Importent Formula

$$\int e^x dx = e^x + c$$

$$\int \cos x\, dx = \sin x + c$$

$$\int \sin x\, dx = -\cos x + c$$

$$\int \sec^2 x\, dx = \tan x + c.$$

$$\int \operatorname{cosec}^2 x\, dx = -\cot x + C.$$

$$\int \sec x \tan x\, dx = \sec x + c.$$

$$\int \frac{f'(x)}{f(x)} dx = \log f(x) + c.$$

$$\int \tan x\, dx = \int \frac{\sec x \tan x}{\sec x} dx = \log \sec x + c.$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left(\frac{x-a}{x+a}\right) + c.$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}.$$

$$\int \tan^2 x\,dx = \int (\sec^2 x - 1)\,dx = \tan x - x + c.$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + c$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + c$$

$$\int \sec x\,dx = \log|\sec x + \tan x| + c$$

$$\int \sqrt{a^2+x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2+x^2} + a^2\log\left(x+\sqrt{x^2+a^2}\right)\right)$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2-x^2} + a^2\sin^{-1}\left(\frac{x}{a}\right)\right)$$

$$\cancel{\int \sqrt{x^2+a^2}}$$

$$\int \sqrt{x^2-a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2-a^2} - a^2\log\left(x+\sqrt{x^2-a^2}\right)\right)$$

Integration by parts

Rule: Integral of the products of two function

= First function × integral of the second function

$- \int$ (derivative of first) × integral of second.

Example: $\int \log x \, dx$

$$= \int \underset{\text{II}}{1} \cdot \underset{\text{I}}{\log x} \, dx = \log x \cdot \int 1 \cdot dx - \int \frac{d}{dx} \log x \times \int 1 \cdot dx.$$

$$= x \log x - x + c.$$

$$\int \frac{e^x \, dx}{1-e^x}$$

Take $1 - e^x = t$

$$\frac{dt}{dx} = -e^x.$$

$$dx = \frac{dt}{-e^x}$$

$$-\int \frac{1}{t} dt = -\log|t|$$

$$= -\log(1 - e^{+x}).$$

(*) $\int \frac{x+1}{x-1} dx$

$$= \int \left(1 + \frac{2}{x-1}\right) dx$$

$$= \int dx \quad x + 2\int \frac{1}{x-1} dx$$

$$= x + 2\log(x-1) + c.$$

$y = \log_e x$

$\Rightarrow x = e^y$

$e^{\log x} = x$.

~~d(x+y~~ $d(xy) = x\,dy + y\,dx$

$d\left(\frac{x}{y}\right) = \frac{y\,dx - x\,dy}{y^2}$

$\frac{d^2y}{dx^2} + y = 0$; find the solution of y?

sol^n:

$y = \sin x$ is a solution.

$$\frac{d}{dx}\left(\frac{d}{dx}\sin x\right) + \sin x = 0$$

$$-\sin x + \sin x = 0$$

A differential equation of the form

$F\left(x, y, \frac{dy}{dx}\right) = 0$ is said to be of first order and first degree

A differential equation of the form

$$F\left(x, y, \frac{dy}{dx}, \left(\frac{dy}{dx}\right)^2, \ldots \left(\frac{dy}{dx}\right)^n\right) = 0$$

$F(x, y, p, p^2, \ldots p^n) = 0$, $p = \frac{dy}{dx}$ is of order one and degree n.

By elimination of the constant c, obtain the differential equation of which

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$$y^2 = 4c(x+c) \quad \text{is the solution.}$$

proof:

$$y^2 = 4c(x+c) \quad \text{--- (i)}$$

Differentiating (1) w.r.t. x

$$\Rightarrow \quad 2y\frac{dy}{dx} = 4c$$

$$\Rightarrow \quad y\frac{dy}{dx} = 2c$$

$$\Rightarrow \text{ put } \boxed{c = \frac{y}{2}\,dy/dx} \text{ in (i)}$$

$$\Rightarrow \quad y^2 = 4c(x+c)$$

$$y^2 = 4 \cdot \frac{y}{\cancel{2}} \cdot \frac{dy}{dx}\left(x + \frac{y}{2}\frac{dy}{dx}\right)$$

$$y^2 = 2xy\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^2$$

$\Rightarrow$ ~~$1 = 2x$~~

$$\Rightarrow y = 2x\frac{dy}{dx} + y\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \left[y - \left(\frac{dy}{dx}\right)^2\right] = 2x\frac{dy}{dx}.$$

Wronskian and its properties

Fundamental existence theorem :-

\# Let $P_0(x)$, $P_1(x)$, $P_2(x)$ and $R(x)$ be continuous function on an interval (a,b) and $P_0(x) \neq 0$ for each $x \in (a,b)$

If c_0 and c_1 are arbitrary real number and $x_0 \in (a,b)$, then there exist a unique solution $y(x)$ of

$$P_0(x) y'' + P_1(x) y' + P_2(x) y = R(x)$$

satisfying $y(x_0) = c_0$, $y'(x_0) = c_1$.

Example :- $y = 3e^{2x} + e^{-2x} - 3x$ is the unique solution of the initial value problem

$$y'' - 4y = 12x \text{, where } y(0) = 4,\ y'(0) = 1$$

Sol^n:

$$y = 3e^{2x} + e^{-2x} - 3x \quad \text{—(1)}$$

$$y' = 6 \cdot e^{2x} - 2e^{-2x} - 3$$

$$y'' = 12e^{2x} + 4e^{-2x} \quad \text{— (ii)}$$

(11)

$$y'' - 4y = 12x \quad \text{(question given)}$$

From (i) and (ii), we have

$$12e^{2x} + 4e^{-2x} - 4\left(3e^{2x} + e^{-2x} - 3x\right) = 12x$$

Thus

$$\Rightarrow 12e^{2x} + 4e^{-2x} - 12e^{2x} - 4e^{-2x} + 12x = 12x.$$

$$\Rightarrow 12x = 12x.$$

Therefore $y = 3e^{2x} + e^{-2x} - 3x$ is a solution of

$y'' - 4y = 12x$, where

$$y(0) = 3\cdot e^{2\cdot 0} + e^{-2\cdot 0} - 3\cdot 0$$

$$= 3 + 1 - 0 = 4.$$

$$y'(0) = 6\cdot e^{2\cdot 0} - 2\cdot e^{-2\cdot 0} - 3$$

$$= 6 - 2 - 3 = 1$$

From Fundamental Existence theorem.

we have $y'' - 4y = 12x$

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$P_0(x) = 1$

$P_1(x) = 0$

$P_2(x) = -4$

$R(x) = 12x$.

all $P_0(x)$, $P_1(x)$, $P_2(x)$ and $R(x)$ are continuous functions in $(-\infty, \infty)$ and $P_0(x) \neq 0$ for each $x \in (-\infty, \infty)$

i,e $y = 3e^{2x} + e^{-2x} - 3x$ is the unique solution of $y'' - 4y = 12x$ with $y(0) = 4$, $y'(0) = 1$

Theorem :- ① Two solution $y_1(x)$ and $y_2(x)$ of the

equation $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$

are linearly dependent if and only if their (13)

wronskian is identically zero.

Examples $W(x) = \begin{vmatrix} y_1(x) & y_2(x) \\ y_1'(x) & y_2'(x) \end{vmatrix} = 0$ on (a,b)

②

Two solution $y_1(x)$ and $y_2(x)$ are linearly independents if and only if their wronskian is not zero at some point $x_0 \in (a,b)$

Example. $\sin x$ and $\cos x$ are linearly independents

solution of $y'' + y = 0$

The wronskian of $y_1(x) = \sin x$ and $y_2(x) = \cos x$

$$W(x) = \begin{vmatrix} y_1(x) & y_2(x) \\ y_1'(x) & y_2'(x) \end{vmatrix}$$

$$= \begin{vmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{vmatrix} = -1 \neq 0$$

Is $y_1(x) = x$ and $y_2(x) = |x|$ are linearly independent on $\mathbb{R}$? (14)

Soln: Yes, Even Wronskian can not applied here, because ~~ff~~ $y_2(x) = |x|$ is not differentiable at $x = 0$

Since $y_1(x) = x$ and $y_2(x) = |x|$

$c_1 x + c_2 |x| \equiv 0$, implies

$c_1 x + c_2 x = 0$ for $x \geq 0$ and $c_1 x - c_2 x = 0$ for $x \leq 0$

and so $c_1 = 0$, $c_2 = 0$.

Homogeneous Equation :-

=) A differential equation of the form

$$\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)} \quad \text{—— (1)}$$

where $f(x,y)$ and $g(x,y)$ are homogeneous function of x and y of the same degree, is called Homo-geneous differential equation.

To solve such an equation, we substitute

$y = vx$ where v is a function of x

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in (1), we can verify that (1) reduces to an equation with separated variable x and v.

Examples :-

$$x \sin\left(\frac{y}{x}\right) dy = \left(y \sin\left(\frac{y}{x}\right) - x\right) dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{y \sin\left(\frac{y}{x}\right) - x}{x \sin\left(\frac{y}{x}\right)}$$

put $y = vx \Rightarrow v = \left(\frac{y}{x}\right)$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow \cancel{x}\frac{dv}{\cancel{dx}} = \cancel{v\,si}$$

$$\Rightarrow \frac{dy}{\cancel{dx}} = \cancel{y}$$

$$\frac{dy}{dx} = \cancel{y \sin}\ \frac{y \sin\left(\frac{y}{x}\right)}{x \sin\left(\frac{y}{x}\right)} - \frac{\cancel{\sin}\left(\frac{y}{x}\right)}{v}$$

$$\frac{dy}{dx} = \frac{v \sin v}{\sin v} - \cancel{\sin v} \cdot \frac{1}{v}$$ (17)

$$\frac{dy}{dx} = v \cdot \frac{\sin v}{\sin v} - \frac{1}{\sin v}$$

$$= v - \frac{1}{\sin v}$$

$$\frac{dy}{dx} = \frac{v \sin v - 1}{\sin v} = v + x\frac{dv}{dx}$$

$$x\frac{dv}{dx} = \frac{v \sin v - 1}{\sin v} - v$$

$$x\frac{dv}{dx} = \frac{v \sin v - 1 - v \sin v}{\sin v}$$

$$x\frac{dv}{dx} = -\frac{1}{\sin v}$$

$$\Rightarrow \sin v \, dv = -\frac{dx}{x}$$

$$\sin v \, dv = -\frac{dx}{x}$$

Integrating both sides, we have.

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$$\Rightarrow \int \sin v \, dv = -\int \frac{dx}{x}$$

$$\Rightarrow \cancel{-\cos x dv} \; -\cos v = -\log(x) + c$$

$$\Rightarrow \boxed{-\cos v + \log x = c}$$

the required solution.

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$y - x\frac{dy}{dx} = x + y\frac{dy}{dx}$.

$\Rightarrow \quad y\frac{dy}{dx} + x\frac{dy}{dx} = x - y$.

$\Rightarrow \quad \frac{dy}{dx}(x+y) = \cancel{x-y}\ y-x$

$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{\cancel{x-y}}{x+y}\right)^{y-x} = \frac{y-x}{y+x}$.

put $y = vx$. $\Rightarrow v = \left(\frac{y}{x}\right)$

$v + x\frac{dv}{dx} = \frac{dy}{dx}$.

$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{\frac{y-x}{x}}{\frac{y+x}{x}}\right) = \frac{\left(\frac{y}{x} - 1\right)}{\left(\frac{y}{x} + 1\right)}$

$\frac{dy}{dx} = \left(\frac{v-1}{v+1}\right)$

$\cancel{v + x\frac{dv}{dx} = v - v}$

$$v + x\frac{dv}{dx} = \frac{v-1}{v+1}. \qquad 20$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v-1}{v+1} - v.$$

$$x\frac{dv}{dx} = \frac{v-1-v^2-v}{v+1}$$

$$x\frac{dv}{dx} = \frac{-(1+v^2)}{v+1}.$$

$$\Rightarrow x\frac{dv}{dx} = -\left(\frac{1+v^2}{v+1}\right).$$

$$\Rightarrow \left(\frac{v+1}{1+v^2}\right)dv = -\frac{dx}{x}.$$

Now integrating both sides, we get

put $1+v^2 = t$

$\Rightarrow 2v = \frac{dt}{dv}$

(21)

$\Rightarrow dv = \frac{dt}{2v}$ $\Rightarrow$ $2v$

$\Rightarrow \int \frac{\cancel{1}}{\cancel{4}}$

$\Rightarrow \int \frac{\cancel{v}\,dv}{1+v^2} + \int \frac{1}{1+v^2}\,dv = -\int \frac{dx}{x}$

$\Rightarrow \frac{1}{2}\log(1+v^2) + \tan^{-1}(v) = -\log x + c.$

$\Rightarrow \frac{1}{2}\log(1+v^2) + \tan^{-1}(v) + \log x = \log c$

$\Rightarrow \frac{1}{2}\log(1+v^2) + \log x - \log c = -\tan^{-1} v.$

$\Rightarrow \frac{1}{2}\log\left(\frac{x\cdot(1+v^2)}{c}\right) = -\tan^{-1} v.$

$\Rightarrow$ required solution.

Non- Homogeneous Equation :

~~Non~~ Non - homogeneous equation of the First degree in x and y are of the Form.

$$\frac{dy}{dx} = \frac{ax + by + c}{Ax + By + c} \quad \text{— } (1)$$

equation (1) can be reduce to the homogeneous Form as follow

put $x = X + h$, $y = Y + K$, where h and K are constant.

$$\frac{dx}{dX} = 1 \quad \text{— } (2)$$

$$\frac{dy}{dY} = 1 \quad \text{— } (3)$$

$$\frac{dy}{dx} = \frac{dy}{dY} \cdot \frac{dY}{dX} \cdot \frac{dX}{dx}$$

From (2) and (3), we have

$$\frac{dy}{dx} = 1 \cdot \frac{dY}{dX} \cdot 1 \cdot$$

$$\boxed{\frac{dy}{dx} = \frac{dY}{dX}}$$

Now equation (1) becomes

$$\frac{dY}{dX} = \frac{a(X+h) + b(Y+k) + c}{A(X+h) + B(Y+k) + C}$$

$$\boxed{\frac{dY}{dX} = \frac{aX + bY + (ah + bk + c)}{AX + BY + (Ah + Bk + C)}} \quad (7)$$

We choose the constant h and k in such a way that

$$\left.\begin{aligned} ah + bk + c &= 0 \\ Ah + Bk + C &= 0 \end{aligned}\right\} \quad (5)$$

(23)

From (4) and (5), we have.

$$\frac{dY}{dX} = \frac{aX + bY}{AX + BY} \quad \left(\frac{a}{A} \neq \frac{b}{B} \right) \quad -(6)$$

From equation (6), it show the behaviours of Homogeneous equation in X and Y which can be solved by the substitution $\boxed{Y = VX}$

Example :- $\frac{dy}{dx} = \frac{x - 2y + 5}{2x + y - 1}$ — (i)

putting $x = X + h$

$y = Y + K$

then equation (i) transform

$$\frac{dY}{dX} = \frac{(X + h) - 2(Y + k) + 5}{2(X + h) + (Y + k) - 1}$$

Example Now choose the h and k so that

$$\frac{dY}{dX} = \frac{X - 2Y + (h - 2k + 5)}{2X + Y + (2h + k - 1)}$$ becomes

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$$\frac{dY}{dX} = \frac{X - 2Y}{2X + Y} \quad \text{--- (2)}$$

ie $h - 2k + 5 = 0$ and $2h + k - 1 = 0$

Equation Behave as a homogeneous equations.

ie $\frac{dY}{dX} = \frac{X - 2Y}{2X + Y}$ is a homogeneous equation.

Now substitute $Y = VX$

So that $\boxed{\frac{dy}{dx} = v + x\frac{dv}{dx}}$

$\Rightarrow$ $\cancel{V = \frac{Y}{}}$ $\boxed{V = \left(\frac{Y}{X}\right)}$

$$\frac{dy}{dx} = \frac{x-2y}{2x+y}.$$

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$$\frac{dy}{dx} = \frac{1-2\left(\frac{y}{x}\right)}{2+\left(\frac{y}{x}\right)}$$

$$\frac{dy}{dx} = \frac{1-2v}{2+v}$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\cancel{x\frac{dv}{dv} = v -}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{1-2v}{2+v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1-2v}{2+v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1-2v-2v-v^2}{2+v}$$

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$$\Rightarrow x\frac{dv}{dx} = \frac{1-4v-v^2}{2+v}.$$

$$\Rightarrow x\frac{dv}{dx} = \frac{(1-4v-v^2)}{(2+v)}$$

Integerating both sides, we have.

$$\Rightarrow \int\left(\frac{2+v}{1-4v-v^2}\right)dv = -\int\frac{dx}{x}$$

$\Rightarrow$ put $t = v^2-4v-1.$

$$\frac{dt}{dv} = 2v-4$$

$$dt = 2(v-2)\,dv.$$

$$\Rightarrow \frac{1}{2}\int\frac{dt}{t} = -\log x + \log c.$$

$$= \frac{1}{2}\log t + \log x = \log c.$$ 28

$$\Rightarrow \log t^{1/2} + \log x = \log c.$$

$$= \log(t^{1/2} . x) = \log c.$$

$$\Rightarrow t^{1/2} . x = c.$$

$\Rightarrow$ squaring both sides, we have

$$t . x^2 = c^2$$

$$t x^2 = c$$

where $t = v^2 + 4v - 1$.

$$\Rightarrow (v^2 + 4v - 1) x^2 = c$$

we know that $v = \left(\frac{y}{x}\right)$

$$\left(\left(\frac{y}{x}\right)^2 + 4\left(\frac{y}{x}\right) - 1\right)x^2 = c$$

(3)

$$y^2 + 4yx - x^2 = c$$

(28)

Now from $h - 2k + 5 = 0$

$2h + k - 1 = 0$

(29)

$\Rightarrow$ $h - 2k = -5$

$2h + k = +1 \times 2$

$h - 2k = -5$

$4h + 2k = 2$

$(-) \quad (+) \quad (+)$

$+5h = -3$

$h = -3/5$

So $-k = \dfrac{-5 - h}{2} = \dfrac{-5 + 3/5}{2}$

$= \dfrac{-25+3}{10} = +\dfrac{\cancel{22}\,11}{\cancel{10}\,5}$

$k = 11/5$

i.e $X = x + 3/5 \quad - (4)$

$Y = y - 11/5 \quad - (5)$ (30)

putting (4) and (5) in equation (3)

we have

$Y^2 + 4YX - X^2 = c$

$\Rightarrow (y - 11/5)^2 + 4(y - 11/5)(x + 3/5)$

$- (x + 3/5)^2 = 0$

$\Rightarrow y^2 - \frac{2 \cdot 11}{5} y + \frac{11^2}{5^2} + 4y - \frac{4 \cdot 11}{5}$

$- \left(x^2 - 2 \cdot \frac{3}{5} x + \left(\frac{3}{5}\right)^2\right) = 0$

$\Rightarrow x^2 - y^2 - 4xy + 10x + 2y = c$

is the required solution.

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Exact Differential Equation

⇒ Exact differential equation is formed by equating an exact differential to zero

* $M\,dx + N\,dy = 0$ where M and N are function of x and y is exact if and only if

$$\boxed{\frac{dM}{dy} = \frac{dN}{dx}}$$

The Solution of the exact differential $M\,dx + N\,dy$ is given by the following :-

(1) Taking y as constant, Integrate M with respect to x

Symbolic :- ⇒ $\int M\,dx$

(2) Integrate those terms of N with respect to y which do not contain x

<u>Symbolics</u> :- $\int (N \sim x)\, dy$

<u>Example</u> :- $(2x - y + 1)\,dx + (2y - x - 1)\,dy = 0$

Here $M = 2x - y + 1$

$N = 2y - x - 1.$

So $\frac{dM}{dy} = -1$, $\frac{dN}{dx} = -1.$

$\Rightarrow$ implies given equation are exact.

$$\int M\,dx = \int (2x - y + 1)\,dx = 2\frac{x^2}{2} - xy + x$$

$$= x^2 - xy + x$$

$\cancel{\int N\,dy}$

$$\int (N \sim x)\,dy = \int (2y - 1)\,dy = y^2 - y.$$

Hence the required solution is

$$\underbrace{x^2 - xy + x}_{\int M dx} + \underbrace{y^2 - y}_{\int (N \sim x) dx} = c \text{ (constant)}$$

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$$\boxed{\int M dx + \int (N \sim x) dx = 0.}$$

Theorem :

If N is not independent of x,

then $\int (N \sim x) dy = 0$

Example : $\left[y \left(1 + \frac{1}{x}\right) \cos y \right] dx + \left[(x + \log x) (\cos y - (\sin y) \cdot y) \right] dy = 0$

$\Rightarrow$ Here $M = y \left(1 + \frac{1}{x}\right) \cos y$

$N = (x + \log x)(\cos y - y \cdot \sin y)$

$$\frac{dM}{dy} = \left(1+\frac{1}{x}\right)(\cos y - y\sin y)$$

$$\frac{dN}{dx} = \left(1+\frac{1}{x}\right)(\cos y - y\sin y)$$

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Here $\frac{dM}{dy} = \frac{dN}{dx}$

So the given equation is exact

$$\int M dx = \int \left(1+\frac{1}{x}\right) y\cos y\, dx$$

$$= y\cos y\,(x + \log x)$$

Now N is not independent of x, i.e N contain all x, so

$$\int (N \sim x)\, dy = \int 0 \cdot dy = 0$$

So the required solution is

$$\boxed{(x+\log x)\, y\cos y = c}$$

Linear equation

⇒ A differential equation of the form

$$\frac{dy}{dx} + Py = Q \quad \text{— (I)}$$

where P and Q are function of x, is called a linear differential equation of first order

Here $e^{\int p\,dx}$ is called integrating Factor

(I.F)

⇒ $y(I.F) = \int Q(I.F)\,dx + c$ is

the solution of equation (I)

Example :- $\frac{dy}{dx} = y \tan x - 2\sin x$

$$\Rightarrow \frac{dy}{dx} - y\tan x = -2\sin x$$

Here $P = -\tan x$

$Q = -2\sin x$. (37)

$\int p\,dx = -\int \tan x\,dx = -\log(\sec x)$

$= -\log(\sec x)$

$= -\log(\cos x)^{-1}$

$= (-1)\times(-1)\log\cos x$

$= \log\cos x$.

$I.F = e^{\int p\,dx} = e^{\log\cos x} = \cos x$

Now the required solution is

$y \cdot I.F = \int Q(I.F)\,dx + c$

$y\cos x = \int(-2\sin x)\cdot\cos x\,dx + c$

$= -\int \sin 2x\,dx = \frac{\cos 2x}{2} + c$

~~$y = \cos x$~~ $\boxed{y\cos x = \frac{\cos 2x}{2} + c}$

Reverse linear equation :-

$$\boxed{\frac{dx}{dy} + Rx = S}$$

$$I \cdot F = e^{\int R\,dy}$$

$$x \cdot (I \cdot F) = \int S \cdot (I \cdot F)\, dy.$$

Example :- $(2x - \log^3)\frac{dy}{dx} + y = 0$

$$\Rightarrow \frac{dy}{dx}(2x - \log^3) = -y.$$

$$\Rightarrow \frac{dx}{dy} = -\cancel{\phi}\frac{1}{y}$$

$$\Rightarrow dy\,(2x - \log^3) = -y\,dx$$

$$\Rightarrow \frac{(2x - \log^3)}{y} = -\frac{dx}{dy}.$$

$$\Rightarrow \frac{+dx}{dy} = \frac{-(-10y^3 + 2x)}{y}$$

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$$\frac{dx}{dy} = \frac{10y^3 - 2x}{y}$$

$$\frac{dx}{dy} + \frac{2}{y} \cdot x = 10y^2.$$

Her $R = \left(\frac{2}{y}\right)$, $S = 10y^2$.

$$I.F = e^{\int \frac{2}{y} dy}.$$

$$= e^{2 \log y} = e^{\log y^2} = y^2$$

Now

$$x \cdot (I.F) = \int S (I.F) \, dy$$

$$x \cdot y^2 = \int 10y^2 \cdot y^2 \, dy.$$

$\boxed{xy^2 = 2y^5 + c}$ is the required solution.

Equation Reducible to the linear Form :-

⇒ Consider a differential equation of the form (40)

$$\frac{dy}{dx} + Py = Qy^n \quad \text{—— (1)}$$

where P and Q are function of x.

we can can reduce the equation (1) to the linear form as given below

Dividing both sides by y^n in equation (1)

$$\Rightarrow \frac{\frac{dy}{dx}}{y^n} + \frac{Py}{y^n} = \frac{Qy^n}{y^n}$$

$$\Rightarrow y^{-n}\frac{dy}{dx} + y^{-n}Py = Q$$

$$\Rightarrow y^{-n}\frac{dy}{dx} + y^{-n+1}P = Q \quad \text{--- (2)}$$

Now put $y^{-n+1} = z$

Differentiate w.r.t x.

$$\Rightarrow$$

$$(-n+1)\,y^{-n}\frac{dy}{dx} = \frac{dz}{dx}. \quad \text{--- (3)}$$

From (2) and (3) we have.

$$\Rightarrow \frac{dz}{dx} \times \left(\frac{1}{-n+1}\right) + zP = Q$$

$$\Rightarrow \boxed{\frac{dz}{dx} + z(1-n)P = Q(1-n)}$$

$$\frac{dz}{dx} + (1-n)zP = (1-n)Q$$ from

a linear equation with z as dependent variables.

Example :-

$$\frac{dy}{dx} - 2y\tan x = y^2\tan^2 x \quad — (1)$$

Now dividing (1) by y^2, we get

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$$\frac{\frac{dy}{dx}}{y^2} - \frac{2y\tan x}{y^2} = \frac{y^2\tan^2 x}{y^2}$$

$$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} - \frac{2}{y}\tan x = \tan^2 x. \quad — (2)$$

Now put $z = -\frac{1}{y}$

$$\frac{dz}{dx} = -\frac{1}{y^2}\frac{dy}{dx} \quad — (3)$$

from (2) and (3), we have